Study Guide

Joint and combined variation
02/29/2012

Joint and Combined Variation
Variation equations are formulas that show how one quantity changes in relation to one or more quantities. There are four types of variation: direct, indirect (or inverse), joint, and combined. This skill focuses on joint and combined variation.

Direct variation equations show a relationship between two quantities such that when one quantity increases, the other also increases, and vise versa. We can say that y varies directly as x. Direct variation formulas are of the form $y = kx$, where the number represented by k does not change and is called a constant of variation. For example, the amount of money in a paycheck (P) varies directly as the number of hours (h) worked. In this case, the constant k is the hourly wage, and the formula is written $P = kh$.

Indirect variation formulas show that when one quantity increases, the other quantity decreases, and vise versa. For example, when the price of an item increases, the demand decreases. Indirect variation formulas are of the form $y = k/x$.

Joint variation formulas show the relationship between a quantity and the product of two other quantities. They are of the form $y = kxz$. For example, the equation for the area of a triangle states that the area is equal to one half times base times height, or A = ? bh. The number ? is the constant of variation, and is always the same for the area of a triangle.

Combined variation formulas show how a quantity varies directly with one or more quantities and indirectly with one or more different quantities. They are of the form $y = (kx)/z$. For example, the sales (S) of a company vary directly as the amount spent on advertising (a), and indirectly as the price (p) of the item sold. The formula is written as $S = (ka)/p$. When the amount spent on advertising increases, sales increase because sales and advertising vary directly. When the price increases, sales decrease, because price and sales vary indirectly.

To solve variation problems, follow these steps.

Step 1: Substitute the quantities given for the letters they represent.
Step 2: Solve for the constant k.
Step 3: Set up the equation again.
Step 4: Substitute given quantities and the value of k that was just determined.
Step 5: Solve for the quantity needed.

Example 1: The surface area (S) of a cube varies jointly with the length (l) and width (w) of one of the faces. The equation that represents this relationship is given by

$$S = klw$$

where k is a constant that needs to be determined. If the surface area of a cube is 150 square inches when the length and width of each face are 5 inches, what are the length and width of each face when the surface area is 216 square inches?

(1) $S = 150,\ l = 5,\ w = 5,\ k = ?$

(2) $150 = k(5)(5)$

$$\frac{150}{25} = \frac{25k}{25}$$

$$k = 6$$

(3) $S = 216,\ k = 6,\ l = ?,\ w = ?$

$$216 = (6)(l)(w)$$

(4) $$\frac{216}{6} = \frac{(6)(l^2)}{6}$$

$$36 = l^2$$

$$\sqrt{36} = \sqrt{l^2}$$

$$6 = l,\ 6 = w$$

<u>Step 1</u>: Write the values for the variables that are known (do not use the values for a surface area of 216 because the constant *k* is not known).
<u>Step 2</u>: Substitute the values into the formula and solve for *k*. Multiply the terms on the right side of the equal sign to get 25*k*. Isolate the *k* by dividing each side of the equation by 25. $k = 6$.
<u>Step 3</u>: Now that the constant is known, it is possible to determine the length and width of a cube that has a surface area of 216 in.^{2}. Write the values for the variables that are known for a surface area of 216.
<u>Step 4</u>: Substitute the values into the variation equation and solve for *l* and *w*. Multiply the right side of the equation to get $6l^2$. Since length and width are the same for a cube, $(l)(w)$ can be written l^2 or w^2. Divide each side of the equation by 6 to isolate the l^2. To remove the square from l^2, take the square root of each side of the equation. $l = 6$ and $w = 6$ (because *l* and *w* are the same).
Answer: length = 6 inches and width = 6 inches

Example 2: The amount of monthly sales (*S*) for a company that makes a certain type of chair varies directly with the amount spent on advertising each month (*a*) and indirectly with the price (*p*) of the chair. The equation that represents this relationship is given by

$S = \frac{ka}{p}$ where *k* is a constant to be determined. If the sales for July were $40,000.00 when advertising costs were $100.00 and the price of the chair was $50.00, what were the sales for August if advertising costs were increased to $135.00 and the price of the chair was decreased to $45.00?

(1) $S = 40{,}000;\ a = 100;\ p = 50$

(2) $$40{,}000 = \frac{k(100)}{50}$$

$$40{,}000 = \frac{100k}{50}$$

(3) $$\frac{40{,}000}{2} = \frac{2k}{2}$$

$$20{,}000 = k$$

(4) $S = ?,\ k = 20{,}000;\ a = 135;\ p = 45$

$$S = \frac{20{,}000(135)}{45}$$

$$S = \frac{2{,}700{,}000}{45}$$

$$S = 60{,}000$$

<u>Step 1</u>: Write the values for the variables that are known (do not use the values for advertising cost of $135 and price of $45 because the constant *k* is not known).
<u>Step 2</u>: Substitute the values into the formula and solve for *k*. Multiply the terms on the top of the fraction to get 100*k*. Divide 100*k* by 50 to further simplify the right side of the equation and get 2*k*. Isolate the *k* by dividing both sides of the equation by 2, yielding $k = 20{,}000$.
<u>Step 3</u>: Now that the constant is known, it is possible to determine the sales for advertising costs of $135 and price of $45. Write the values for the variables that are known for these constraints.
<u>Step 4</u>: Substitute the values into the variation equation and solve for *S*. Multiply the numbers on the top of the fraction to get 2,7000,000. Divide 2,700,000 by 45 to continue to simplify the equation and determine that the sales were $60,000.00
Answer: $60,000.00

An activity that can help reinforce the concept of variation is to describe situations involving joint and combined variation, and then have the student write the equations relating the quantities. For example, "The volume of a cylinder varies jointly as the square of the radius of the cylinder and its height." The student would write: $V = kr^2 h$.